

# Vibrational Characteristics of Bell Plates

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## Abstract:

Two thin aluminium plates of specific shape – a composite rectangle/equilateral triangle and a simple equilateral triangle – have been found, when constructed with and held at handles, to radiate a steady state and virtually pure tone, when struck. This property, along with the capacity to manipulate the amplitude, onset and decay of the tone, make the plates suitable for use in musical performance, with the composite shape being used in the manufacture of pitched sets of *Belleplates*®.

Using experimental and computational techniques, these handheld instruments, collectively known as *bell plates*, were studied here to determine the possible origin of their tone-producing mechanisms. Their mode shapes and vibrational dynamics were compared with those of two non-ringing plates, whose dimensions vary only slightly from those of the bell plate group. The shapes of these non-ringing plates – here termed *transient bell plates* - are a composite rectangle/isosceles triangle and a simple isosceles triangle. As with the bell plates, they are held at stems or handles.

Representations of the shapes of the lowest vibrational modes of the four ringing and non-ringing plates were obtained using *Chladni patterns*, in which sand grains

are used to highlight any nodal regions occurring in a given mode. The mode shape of the ringing mode in the bell plates was identified as an unbroken nodal line in the shape of a U-curve. This curve separates a central oscillating region from two symmetrically-positioned outer regions oscillating out-of-phase with the central area. The mode shape of the equivalent mode in the transient plates is similarly divided into central and outer regions, but these are not divided by a single unbroken curve. Instead, the central *antinode* in the equivalent mode of the transient plate is outlined by two straight lines on either side of the central axis.

The mode shapes found in the two bell plates and the two transient plates were then verified, and their dynamics analyzed, by the use of *Finite Element Modelling* (FEM). The FEM results of this research show that the ringing mode dominates the vibrational spectra of the two bell plates, verifying the almost pure-tone characteristics of these plates. The spectra of the two transient plates are also dominated by a single mode, which in those plates does not radiate acoustically. With further FEM analysis, features were discovered which differentiate bell plate modal dynamics from those of the non-ringing plates, highlighting characteristics of possible tone-production mechanism in the former.

In the ringing mode of the bell plates, greater deformations around the horizontal and central vertical axes were found compared to those in the equivalent transient plate modes. Strain energy density, concentrated at the top centre of the bell plates in this mode, is clearly more dissipated across the area of the transient plates. An orderly alignment of out-of-plane displacement vectors in the belleplate is absent in its non-ringing counterpart. The value of Poisson's Ratio – a measure of the coupling of perpendicular flexural motions in a material – is found to be critical to the presence of the ringing mode's unbroken U-curve but not to that of the broken nodal lines in the transient plates.

These comparisons highlight certain features of bell plate vibrational dynamics which are believed to be characteristic of effective tone-generation mechanisms. Future experimental and computational work could reveal further qualitative and

quantitative characteristics in both plate types, thus extending and refining an understanding of their significant differences.

## **CERTIFICATE OF AUTHORSHIP/ORIGINALITY**

I certify that this thesis has not previously been submitted for a degree nor has it been submitted as part of requirements for a degree except as fully acknowledged within the text.

I also certify that the thesis has been written by me. Any help that I have received in my research work and the preparation of the thesis itself has been acknowledged. In addition, I certify that all information sources and literature used are indicated in the thesis.

Signature of Candidate

Dharan

## **Acknowledgments**

I wish to especially thank the following people:

My supervisor, Suzanne Hogg, for her kindness, understanding and tireless patience over the whole course of this project, for many years of helpful discussions and for her assistance in the final preparation of this document.

Professor Joe Wolfe of the Musical Acoustics Group at the University of New South Wales, for kind and generous advice, for the use of the Musical Acoustics Lab to produce the Chladni patterns shown in Chapter Three, for invaluable discussions of the importance of Poisson's Ratio, and for suggesting the modelling of Fig 3.12.

Dr Tony Fischer-Cripps and Dr Suzanne Thwaites of the CSIRO Division of Telecommunications and Industrial Physics, for their interest and support and for valuable discussions of the nature of the Finite Element Modelling results.

Mr Terry Brown of the UTS Faculty of Engineering for his interest and support, for valuable lecture notes on the principles and foundations of FEM, and for permitting me to attend his class lectures dealing with this topic.

Mr Daniel Schiavazzi of G+D Computing, for valuable technical assistance with the Strand 7 Finite Element Analysis software.

Mr Jim Franklin of the UTS Department of Applied Physics for many years of fruitful and enjoyable discussions.

The academic and technical staff of the Applied Physics Department for a pleasant and encouraging work environment.

My many friends and relatives for their constant interest and support over the years.

My parents, without whose faith, understanding and loving kindness this project would never have been completed.

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